By using an algebraic approach, Hansson [1990] suspected that something was very wrong about our measure of freight transport on lorries in the national accounts. Hansson [2000a][2000b][2001] emphasized and verified that suspicion, and argued that the measure during the 1990s badly have going worse, indicating a quite misleading picture to our users. One of the main reasons for this miscalculation is the use of a kind of price index for outlays, such as petrol, wages, tires, telephone, interests, and other outlays of intermediate and financial character when deflating current production incomes from the freight transportations. Such a convention can be regarded as acceptable if the correlation coefficient is rather close to +1 between that cost price index and the current income price index. According to Hansson [2001] this is far from the case, rather close to the opposite, i.e. -1. If that is entirely correct, then we should not recommend using our figures in sophisticated model applications, neither in serious and meaningful business cycles analyses, in or economic forecasting, or in formulating and stating economic policy. Here we are in urgent need for a more accurate quantity and price indices, based upon prices and quantities of traded service productions that correspond with each other. How it looks like is shown in Diagram 6 and Diagram 7.

But the problem is deeper than that. Fright transports on lorries are a rather complex service. First it can be regarded as combined service produced by a forwarding agent and its contracted haulage contractors. And it is this combined service the customers have to pay for. So here we can talk about three distinct but inter correlated payments, the clients' payment for the specific transports, and their split between the forwarding agent and the haulage contractors. And second there are no unique and lasting service within these payments flows, but numerous and all the time changing. And in almost every occasion the different unique services have many dimensions. The whole complexity is more detailed described in Hansson [1996]. The complexity is of such a degree that a conventional index compilation, like the one discussed and suggested by Eurostat [2001], hardly can capture it in a satisfactory manner. In fact it is not possible at all, especially regarding the analytical criteria users normally raise.

1 The structure of commercial freight transports on road

When you more closely regard the present Swedish compilation of the production value in current and fixed prices in freight transports on road, it turns out to be a very crude measurement. For several years, and independently, income figures are gathered in current prices from the forwarding agents and the haulage contractors. To get fixed prices those independent figures are deflated with a kind of cost price index. Such a measurement will be correct if the income and costs not only are equal, but also that the their price indices are equal. This is far from the case. For example between the income and costs a certain surplus can be observed, which can be of a positive as well as a negative value. Moreover, there are definite signs, as shown in Hansson [2000a][2000b][2001], that the correlation between the production price indices and the used cost price index are strongly negative, at least since the middle of 1990s. Also the price development between freight price and its split between forward agent price and haulage contractor price are quite different. Furthermore, a close inquire reveal that a compilation like the present schablon of indicator will not do, instead a rather detailed measurement have to be settled. There are a number of determinant reasons for that.

Commercial Freight transport on roads is a complex and many dimensional services. A customer, or client, orders a freight transportation between the geographical points *A* and *B* from a forwarding agent. The forwarding agent hand over the order to a haulage contractor, or several haulage contractors, who actually do the freight transportation. The customer pays for the freight to the forwarding agent, who split the payment between him and the haulage contractor.

There are two ways to look upon the described logistic. The conventional one is to regard it as consisting of two different services, the service of forwarding agents and the service of the haulage contractors. There is one troublesome drawback with this approach. When you statistically measure these services separately, you loose the natural and close intermediate and simultaneously connection between them, which at least create inconsistencies in the national accounts. In that matter it is safer to regard freight transportations as a combined services in many dimensions. Such an approach is more fruitful and effective. For example, you can all the time withhold the intermediate relation needed for the national accounts. Consequently, in the following I will regard freight transport on roads as a *composite service*, consisting of *Freight forwarding on road* and *Road haulage*.

The customers do not pay for these services in separate, but in one sum for the whole freight according to a price list or a special agreement, where the two parts is not separated. It is the forwarding agent who does the separation between the two sub services, when he share the payments with his contracted haulage contractors. To maintain consistency in compilation, it is therefore recommendable to compile the total payments and the different parts simultaneously through the forwarding agents accounting systems. It is also important to be very keen about the compilation of the three different payments in current and fixed prices. The determinant reason is that the structure and number of services considerable are changing all the time. Furthermore, it is possible through the forwarding agents to branch classify the clients.

The customers are subjects like firms, shops, official units, and private persons. The forwarding agent set up and administrates the logistic, like transportations schedules, structure of transportation, policy of pricing, settle agree-

ments with its customers and haulage contractors, administrate the payments and its split. For the freight transportation the forwarding agent contract haulage contractors, which usually then have to use the forwarding agents logotype. A haulage contractor can be a single person with one or two lorries, or a company with a number of lorries. Usually a haulage contractor employs or hire one or more drivers, especially the companies.

There are a number of logistic solutions how the transportation is done. One logistic is that a haulage contractor transports the goods from *A* to a central terminal nearby *A*. Now another haulage contractor get the goods and transport it to a central terminal nearby *B*, where a final haulage contractor deliver the goods to *B*. At least three haulage contractors are involved. But between *A* and *B* several can have an agreement to share the responsibility, which is rather common. Another type of logistic is that the transportation is done directly between *A* and *B* without transshipment via terminals. Also here several haulage contractors can be involved.

What the customer shall pay is determined by a fixed price list or an agreement. Usually the forwarding agent set up a new price list once a year valid from January 1; out of which special agreement is settled and constantly renegotiated. A common feature is that the price is increasing after the New Year eve, and decreasing during the year. The service structure is not necessarily fixed. It can change between years. Also an entirely new price strategy can be introduced. It also happens that the price list is revised during the running period. There are examples of very massy and complicated pricing structures causing heavy administration costs for all parts involved. But the present tendency today is towards more simple and easily handled ones. During the 1990s a number of forwarding agents have completely changed their policy of pricing, especially is that valid for the dominating ones. None of the forwarding agents practice the same kind of pricing policy. Pricing is one of the means in their competition with each other about market shares.

The fact that there is no single price policy valid for all, but a number of quite different and all the time changing, is one of the main reasons why it is not possible to determine a sample of representative freights for the whole branch. Any try in that direction will be a complete failure in all respects, not the least yielding wholly misleading economic statistics. The same is true for any try through hedonic or some kind of model pricing, which will be just a failure. In the following I will come to another determining one, the continually changing mix of traded services, and that there are no unique way of pricing.

A common policy of pricing is that the price list is constructed after weight and distance. Often there are different payments regarding the kind of goods, like fragile, glasswork, gas, and robust cargo. Another structure is that the payment is connected to number of boxes, containers, lorries and the like. Ten tons of goods to be freighted ten km are one unique service. Ten tons of goods to be freighted twenty km are another unique service. If the forwarding agents in addition care about fragile, robust, and carefully packed goods, then you have a number of more unique services, like for example ten tons of goods of fragile goods carefully packed to be freighted ten kilometers, i.e. a service composed of four dimensions, or parameters. If for certain cargos you pay half the price for the part over ten tons, then you have two unique services, one up to ten tons, and one over ten tons, creating a fifth dimension. When you come to accurate and authentic weights in the quantity and price measurement, it is of importance to be rather keen about all these dimensions. And observe, there is no unique pricing policy, neither for the forwarding about his customers, nor between the forwarding agents.

Payments determined by a special agreements with single customers are to its structure mostly the like, but far more complicated as they are continuously changing. All the time customers negotiate what to pay and how. Often this is done under a given price list determined by the forwarding agent. In these cases there are a basic price list out which there are numerous price lists; one or more for each customer with different price structures. And these price lists, as a whole, is constantly changing. There is also number of customers having a special agreement, with no connections to the forwarding agents basic pricelists. Especially is that the case when a company will have an open freight line. They freight goods a lot, but do not know exactly how much and when. Here one agreement can be that a lorry go between *A* and *B* every evening, and for that the customer pay 100 000 a month. Another agreement can be that he pay 30 00 per lorry on call regardless how much is loaded. Another agreement can be that one or more lorries stand by in case.

In all possible cases, if you pay from a price list, or under a special agreement, it is possible to come down to a unique service, either in one or several dimensions. To get the accurate weights for the price and quantity index, it is crucial to compile the indexes at that unique micro level. It is also important to do the compilation daily, as the weight structure heavily changes over the calendar year, despite that the structure of the price list do not change. This is a heavy argument not in favor for a selection of set of representative branch services. It is also a heavy argument against selecting a certain day, like the midday of the month, for price estimation.

The payment for the transportation is split between the forwarding agent and the haulage contractor through the accounting system of the forwarding agent. There are several models how this split is done.

1) The forwarding agent gets his share by a certain per cent out of the freight payments, which might be negative. The haulage contractor gets what its left.
2) For a number of transportations the reverse is by the hand, i.e. the haulage contractors get their compensation by an agreement, which can be a lump sum. The forwarding agent gets what its left, which might be negative.
3) The forwarding agents get their compensation by a number of fees. Common forms are pallet fees, transhipment fees for number of items transshipped in terminals, storage fees, and freight margin above the compensation

to the haulage contractor. These kinds of fees are deducted from the haulage contractor's payment. The haulage contractor is paid under a yearly agreement, or a lump sum. The yearly agreement can be changed during the running period.

4) The haulage contractor is paid according to an agreement. The forwarding agent set his share on top, which can be positive as well as negative, i.e. the forwarding agents takes what its left, which seems to be a rather common model during the 1990s, which indicate that some forwarding agents might have accounted a negative value added for some years.

5) The forwarding agent set a freight margin on top of the compensation to the haulage contractor.

Commonly the haulage contractor's compensation is under an agreement, which usually runs for a year from January 1, but might be changed during the running periods. Mostly a unique haulage service is determined by a number of dimensions, or parameters. The forms of compensation are numerous. In analogy to the case for the price list above, the agreement defines a number of unique haulage services. Common forms of compensation bases are payments per pallet, per cargo, per container, per day, per hour, per month, per ton and kilometres, per unit of freight, and the like. There are also a number of additions, like addition for trailer, addition for certain kinds of dangerous and fragile goods, addition per number of axles, and the like.

The whole logistic is hold together administratively by the *freight order*, which contain all information needed for the freight and its taxation, the payment split between the forwarding agent and its haulage contractors involved, the invoice compilation, and the forwarding agents internal book keeping and periodically financial reports. From the freight order it is also possible to set up a structure for an authentically compiling of the price and quantity indices with the authentically and needed weights for the index numbers.

The crucial character of freight transportations is that it is a service of many dimensions and constantly changing, such that a vast number of new services occur while a vast number are disappearing. Say that a for a certain forwarding 5 % new services are introduced every month. In 12 month the stock of services will increase by 80 %. Say that 2 % will disappear every month. In 12 month the stock of services will decrease by 22 %. Netted in this example, the stock of services will increase by 44 % for that particular forwarding agent.

In such a situation, and the like, a conventional price and quantity compilation, like a representative sample measured at a certain monthly date, will just be a failure during the running year. After a while, one tries to measure large parts that not anymore are traded, while not measuring large parts that are traded. It is impossible to select a statistical meaningful sample of representative freights under such circumstances. Of course you can select some kind of a sample, and call it representative. But in a while you may not have anything left to measure for that particular haulage contractor, at least to a great extent distorted. If you look upon the whole branch, there are neither identical price lists nor contracts or special agreements between the acting forwarding agents. The pricing do not only differ in content, but also in structure and policy. This means that a representative sample on branch level is not a relevant or a proper issue. Every try to design such a sample will immediately be a failure.

Another method used is the so-called model pricing, which mean that one set up some kind of fictitious service and ask what to pay for that. Model pricing cannot reflect an accurate situation. One set up a kind of plausible service, one belief, and asks what you have to pay for it. This service is not traded, and will not be, which mean that the respondent can say what price he likes. The price you get is not determined under a true competitive situation, where it will be lower. In a real negotiation the price will be quite different, and probably not realized if there are a number of firms involved in the negotiation. To what extant the model price will reflect a real situation, we cannot say. But it will be a poor and shaky estimate anyway. Furthermore, by which criteria shall you construct a bunch of model services, to be sure you get a set of "trips" that truly reflect the traded services performed in the haulage branch. I belief it is not a realistic possibility and doomed to fail.

The only fruitful approach left, is to, if it is possible, measure the entirely freight for each forwarding agent involved in the measurement, i.e. the payments from the customers, and its split between the forwarding agent and contracted haulage contractors. As conventional used index forms, especially the so called classical ones, is not designed to meet a changing mix of traded goods and services, the main question is how to design an index suitable for measuring changes in price and quantities in a dyna-mic situation. In Hansson [1996][1998][2000a][2000b], I argue that a weight expounded Laspeyres index is a possible approach left. And in Hansson [2000b] I discuss that one has to concentrate upon an expounded Laspeyres quantity index and let the price index be the expounded Paasche index under the value index.

In a way, I argue in Hansson [1996], we must handle the introduction of new products and its disappearances of other traded products properly in the indexes. A representative sample will not do. And further, such a sample cannot be defined. To be sure a total daily compilation is the only solution that will do. But in that case a number of analytically difficulties of index mathematically kind arise. In Hansson [1996] I discuss a way out of the dilemma of continuously change in traded structure. And Hansson [2000a][2000b][2001] shows that this kind of solution in operation is effective and satisfactory, giving accurate economic figures characterized by a kind of authentic price and quantity measure. In the next section I will heavily rely upon Hansson [2000b].

2 Two approaches when constructing indices

A number of approaches exist to construct an index number. Diewert [1993] notes the fixed basket, statistical, test, divisia, and economic approaches. One can also note the

hedonic technique. None of these will yield accurate data for model applications. I will here consider the test approach, which also likely is called the axiomatic approach, and discuss what kind of criteria that will yield required and meaningful data for accurate and deep economic analysis. By meaningful data I mean such data that cause least possible statistical trouble in scientific sense in model applications.

Under the axiomatic approach, index theorists try to determine a number of desirable criterions for an index number and by help of these formulate the ultimate index. The most famous collection of test is the so-called Fisher's test from 1922, by which he chose what he called the king of indices, more known as Fisher's ideal index. To honor the contribution of Bowley, I will now on call that index for the Bowley-Fisher index.

Over the years Fisher's tests have been admired and questioned. Several persons claim that they are inconsistent by formulating what they call inconsistency theorems. Several persons have also tried to propose another collection of tests, which they regard as more appropriate and more consistent. But to my concern, all so far have failed in that some of mission impossible.

Obviously there is a big, unsolvable, confusion among the index theorists themselves how to regard and handle these sets of axioms, or tests, in strict mathematical sense as well in factual index construction and compilation. I have a growing feeling that people engaged try to verify, maybe in a desperate manner, their own views by a subjected chosen set of axiom[1]. I doubt about the effective usefulness of these axioms in constructing effective statistical estimates in the conventional sense. Such a use has to be thoroughly proved. My opinion, when you come to pragmatic and meaningful application, is that through these kinds of axioms it is not possible to construct any price and quantity index that will be applicable in a serious model application what so ever. For that, I will argue, an entirely new and different philosophy for settlement of desirable criterion is needed, based upon pure pragmatic, mathematical, and econometrical considerations. A proposal to such an approach is described and discussed in section 4.

3 About the classical index theory

The earliest known interest in prices might be from Hammurabi (1792-1750 BC), who legislated against inflation. You can read about it at the Louvren in Paris from the original document, which is a huge black stone, where you also can read about what medical fees and wages that had to be paid all over the vast Mesopotamian Empire. Since then the disaster of inflation has been notated, like in ancient Athens and Roman Empire, the feudal Europe, and the German inflation in the 1920s.

3.1 William Fleetwood's basic index approach

An early example of a price index, according to Diewert [1993], is from William Fleetwood (1656-1723), Bishop of Ely[2]. He estimated what an Oxford student must pay for 5 quarters of wheat, 4 hogsheads of beer and 6 yards of cloth in 1707 and in 1460[3]. Another early example of such an index comes from the US war of independence. To compensate for massive war inflation, the Massachusetts Legislature in 1780 indexed what to pay the soldiers. They set the basket at 5 bushels of corn, 68 and 4/7 pounds of beef, 10 pounds of sheep's wool, and 16 pounds of sole leather.

Fleetwood's index idea has been formulated as,

$$(1)\quad P_{Fl} = \frac{p_2 q}{p_1 q} \qquad \begin{matrix} \textit{(Fleetwood 1707)} \\ \textit{(Lowe 1823)} \end{matrix}$$

where $q = (q^1, q^2, ..., q^n)$ is fixed quantity vector for n chosen goods, and $p_t = (p_t^1, p_t^2, ..., p_t^n)$ is the corresponding price vector in period $t=1,2$. By that one can say, the main feature of a price index was settled, i.e. what to pay for a fixed basket of goods in two different periods. This index idea is also recognized as the fixed basket approach or the tabular standard.

Joseph Lowe 1823 developed the concept in such detail that Diewert [1993] called him the father of the price index. And Studenski [1958, 107] say that Lowe [1823] also was the first calculation of the national income in current and

[1] Following quotation from Balk [1995] might be significant. "Parallel with the invention of new price indices went the development of criteria for distinguishing between them. This was a rather natural process. Inventing a new formula is not enough. One should also provide evidence that the newborn index is bet-

[2] William Fleetwood, English bishop and reformer, was born on 1 January in the Tower of London and educated at King's College, Cambridge. He became one of the most celebrated preachers of his day, often speaking before the Royal family and to the parliament. On 2 June 1702 he was appointed to a canonry at Windsor. He also held other appointments and a fellowship at Eton. He was created Bishop on Ely on 19 November 1714. Many of his sermons were published. He his known for his argument, because slaves are necessary for trade, slave obedience and planter authority should be reinforced by the church. He was also condemned by Parliament for his attacks on Tory principles, which nevertheless was published in The Spectator. In what sense he was a reformer, I do not know, neither what Tory critique was about. William Fleetwood died at Tottenham, near London on 4 August 1723.

In 1707, in London, Fleetwood anonymously published a bound manuscript of 69 pages, called *Chronicon preciosum, or an account of English gold and silver money; the price of corn and other commodities.* Fleetwood addressed the question about the ability to retain a College fellowship while in the possession an estate of practically no value because of the change in the value money.

[3] Diewert [1993] mention that a good account of Fleetwood's contributions with extensive quotations can be found in Ferger [1946].

constant prices. He did that in 1792 prices for 1792, 1806, 1814, and 1823 regarding Great Britain and Ireland[4].

If you are interested to compare what to pay for a certain standard, or to compensate a certain social group, (1) will perfectly do. But in measuring changes in quantities and common prices there are a number of drawbacks with the approach, especially in dynamic situation, where it may not satisfactory work at all.

According to Diewert [1993], a host of people, endorsed (1), such as Jevons [1865], Sidgewick [1883], Marshall [1887], Edgeworth [1887], Bowley [1899], Walsh [1901], and Pigou [1912]. But (1) was considered a problem, and among all Laspeyres and Paasche proposed the well known and all over used (3) and (4), which started a debate that certain averages of these should be used instead, such as (5) - (8).

3.2 Some well known index formulas

Since Lowe's index proposes in 1823 a number of different index forms have been presented. The most famous price formulas, to which we today also consider there exist an analogue quantity formulae, are:

(2) $P_J = \prod_i^n \left(\frac{p_2^i}{p_1^i} \right)^{\frac{1}{n}}$ *(Jevons 1865[5])*

(3) $P_L = \frac{p_2 q_1}{p_1 q_1}$ *(Laspeyres 1871)*

(4) $P_P = \frac{p_2 q_2}{p_1 q_2}$ *(Paasche 1874)*

(5) $P_{S,B} = (P_L + P_P)/2$ *(Sidgewick 1883) (Bowley 1901)*

(6) $P_M = \frac{\sum_i p_2^i (q_1^i + q_2^i)/2}{\sum_i p_1^i (q_1^i + q_2^i)/2}$ *(Marshall 1887) (Edgeworth 1887)*

(7) $P_{B,Fi} = \sqrt{\frac{p_2 q_1}{p_1 q_1} \cdot \frac{p_2 q_2}{p_1 q_2}}$ *(Bowley 1899) (Fisher 1921) (Pigou 1932)*

(8) $P_W = \frac{\sum_i p_2^i \sqrt{q_1^i q_2^i}}{\sum_i p_1^i \sqrt{q_1^i q_2^i}}$ *(Walsh 1901)*

(9) $P_{Pi} = P_P P_L$ *(Pigou 1912)*

(10) $P_T = \prod_i \left(\frac{p_2^i}{p_1^i} \right)^{\left(\left(\frac{p_1^i q_1^i}{\sum p_1^i q_1^i} + \frac{p_2^i q_2^i}{\sum p_2^i q_2^i} \right) \cdot \frac{1}{2} \right)}$ *(Törnqvist 1936)*

(11) $P_S = \frac{\left(\frac{p_1 q_0}{p_o q_0} - \frac{p_o q_1}{p_o q_0} \right)}{2} + \sqrt{\frac{\left(\frac{p_1 q_0}{p_o q_0} - \frac{p_o q_1}{p_o q_0} \right)^2}{2^2} + \frac{p_1 q_1}{p_o q_0}}$ *(Stuvel 1957)*

Under the so called index axioms, index (7) is the Fisher's ideal index. Fisher [1922] named it *the king of indices*. From the same axioms, Stuvel regarded (11) as the superior and outstanding index. Nowadays (8) is regarded as the superlative index form. And recently, the 1997 Swedish

---

[4] Lowe used his national income figures to calculate the burden of the existing taxation. He was especially interested in combating Malthus' theory, or its crude interpretation, that the resources of any country are limited and that an increase in population inevitably brings in its wake a reduction in the standard of living. From his figures Lowe contended that England's volume of production has increased since 1792 at least as rapidly as population.

Lowe projected his figures about population and income to the year 1850. He also projected the population of France to the same year, and concluded that *even as to population, we shall overtake our ancient rival ere another generation pass away*. Particular Lowe was concerned over the increase of burden of the public debt on the shrinking money value of the national income, and proposed the government to enact a tabular standard that would control the value of money and payments of capital income, interest payment on government debt and labor income.

According to Schumpeter [1954, p 526], Lowe was not first to make such a suggestion. Schumpeter points out that Sir George Shuckburgh Evelyn had made essential the same proposal in a paper he presented to the British Statistical Society in 1798, in which he used an index number of a primitive kind for measuring the depreciating of money. Schumpeter states that Lowe added nothing to the idea of tabular standard beyond Evelyn. And he means that Lowe was very unfair to Evelyn's pioneering work.

Schumpeter also stated that there is no excuse for the hesitation with which fact-presenting economists took to the use of price index numbers or theoretical economists to the task of providing a theory for them. Early economists even failed to avail themselves of the most primitive devices for presenting figures.

[5] This kind of index is also referred to the so called Statistical Approach, which according to Diewert [1993], was originated by Jevons [1865][1884] ... proposed that the ... in the ... ply of money increased all prices proportionally except for random fluctuations. The idea with the index form was that with additive errors and sufficient number of independent observations, an appropriate price index could be obtained by taking the arithmetic mean of the price ratios, while with multiplicative errors an appropriate price index could be obtained by taking the geometric mean of the price ratios. According to Diewert the approach was criticized by Irving Fisher [1911, pp 194-196], who in an absolutely convincing manner explained why all prices cannot move proportionally, for example due to the existence of fixed price contracts. But, as Diewert tell us, the profession ignored Fisher's criticism, as those of Walsh [1924].

However, Diewert says, Keynes [1930, pp71-81] effectively demolished the approach by showing that prices did not change proportional, and Keynes used a rather rude language, like the following quotation: *I have long believed that this is a will-o'-the-wisp, a circle-squaring expedition which has given an elusive taint, difficult to touch or catch, to the theory of price index numbers traditional in England. This is not equally true in America. Nevertheless, whilst the Americans have not worshipped the mythical creature, they have not (with the exception, perhaps, of Mr. Walsh) actively combated him or dragged him out of the twilit cave where Edgeworth judiciously kept him.* To fully understand that comment, one have to read about the non scientific quarrel between Walsh and Edgeworth. For the moment I have ... to ... to ...

Consumer Price Committee, SOU [1999], investigated (3), (7), (8), and (10), and proposed a yearly chain index of (8), and month-to-year index of (3).

Nevertheless, none of these index forms should be used in price and quantity measure of freight transport on roads for the national accounts. The same goes for the bunch of indices presented by Irving Fisher in his voluminous book *The Making of Index Numbers* from 1922.There are two main reasons for that. None of them is very useful when measuring a dynamic basket. And none of them can yield authentic measures for meaningful model applications. Before I am going into that discussion more in detail, I will first a little discuss some of the desirable properties a number of persons have formulated under the so-called axiomatic or test approach. I will also concentrate a little upon the so called classical index by pointing out some properties that will lead forward to a more pragmatic approach and what index structure we are looking for. And it turns out that a kind of weight expanded Laspeyres index will do as a start.

3.3 The axiomatic approach

Parallel with the suggestions of suitable or preferable index forms, a firm interest occurred to formulize the index determination by using in a way certain ideas that has shown to be that successful in pure mathematics, like mathematical axioms for a single number and mathematical laws for vectors, upon complex index constructions. According to Diewert [1993] the origins of this attempt, which he call the test approach, are rooted in the more or less casual observations of the early workers of index numbers on their favorite formulae, or those of their competitors. And since Jevons [1865], according to Balk [1995], index theorists have looked for the ultimate index. In that chase Walsh [1901], Fisher [1922] and Eichhorn & Voeller [1976] have proposed a certain number of criteria that an ideal index formula has to stand.

Index theorists like to regard the axiomatic approach as a flourishing branch of the theory of price and quantity indices, like for example Balk [1985, p 59]. And he notate that the research has taken two directions: 1) studying the internal consistency of sets of those axioms, and 2) trying to characterize particular price index formulas by means of sets of those axioms. Balk argues that the second direction continuously yields more or less remarkable results. But I consider that a somewhat exaggerated conclusion. The issue is far from settled. I doubt it ever will be possible to reach any fruitful results for pragmatic index compilation. For that a totally alternative approach is needed.

Thanks to Eichhorn & Voeller [1976], according to Balk [1995], one today distinguishes between self-evident axioms, and still debatable tests. To see the flavour of this kind of idea of the axiomatic approach, let $P(\cdot)$ and $f(\cdot)$ be real valued functions. Balk selected the six following axioms

A1. Monotonisity Axiom

$$P(p_o, p_2, q_o, q_1) > P(p_o, p_1, q_o, q_1) \quad \text{if } p_2 \geq, p_1)$$

$$P(p_o, p_2, q_o, q_1) < P(p_o, p_1, q_o, q_1) \quad \text{if } p_2 \leq, p_1)$$

A2. Linear homogeneity Axiom

$$P(p_o, lp_1, q_o, q_1) = lP(p_o, p_1, q_o, q_1) \quad \lambda>0$$

A3. Identity Axiom (*Laspeyres 1871)*

$$P(p, p, q_1, q_2) = 1$$

A4. Homogeneity of degree 0

$$P(lp_o, lp_1, q_o, q_1) = P(p_o, p_1, q_o, q_1) \quad \lambda>0$$

A5. Dimensional invariance Axiom(commensurability)

$$P(\Lambda p_o, \Lambda p_1, \Lambda q_o, \Lambda q_1) = P(p_o, p_1, q_o, q_1)$$

Λ= real valued vector

A6. Proportionality Axiom *(Walsh 1901)*

$$P(p_1, lp_1, q_1, q_2) = l$$

According to Balk, Eichhorn & Voeller [1976] selected A1-A5, but I can only find A1, A2, A3 and A5, which they consider as indispensable. Balk also means that A2 and A3 imply A6. But then A6 cannot be an axiom; indicating confusion what an axiom here is about. Several other criteria or tests have been proposed in literature.

T1. Base invariance test *(Jevons 1865)*

$$\frac{P(p_p, p_t, q_o, q_t)}{P(p_p, p_s, q_o, q_s)} = \frac{P(p_i, p_t, q_i, q_t)}{P(p_i, p_s, q_i, q_s)}$$

T2. Circularity test (or transitivity) *(Westergard 1890)*

$$P(p_1, p_2, q_1, q_2)P(p_2, p_3, q_2, q_3) = P(p_1, p_3, q_1, q_3)$$

T3. Time reversal test *(Pierson 1896)*

$$P(p_1, p_2, q_1, q_2) = 1/P(p_2, p_1, q_2, q_1)$$

T4. Constant quantity test *(Walsh 1901)*

$$P(p_1, p_2, q, q) = p_2 q / p_1 q$$

T5. Multi period identity test *(Walsh 1901)*

$$P(p_1, p_2, q_1, q_2)P(p_2, p_3, q_2, q_3)P(p_3, p_1, q_3, q_1) = 1$$

T6. Product test *(Fisher 1911)*

$$P(p_1, p_2, q_1, q_2)Q(p_1, p_2, q_1, q_2) = p_2 q_2 / p_1 q_1$$

T7. Factor reversal test

$$P(p_o, p_1, q_o, q_1)P(q_o, q_1, p_o, p_1) = p_1 q_1 / p_o q_o$$

T8. Value dependence test *(Van Yzeren 1952)*

$$P(p_o, p_1, q_o, q_1) = f(p_o q_o, p_1 q_1, p_o q_1, p_1 q_o)$$

T9. Consistency in aggregation test *(Vartia 1974, 1976)*

T10. Determinateness test *(Fisher 1922)*

Fisher's original tests are A5, A6, T2, T7, and T10. Swamy [1965] means that T7 and T10 was suspect. Furthermore, Swamy argues that T2 gives A3 and T3. If so, A3 is not an axiom, and instead might consider T2 as an axiom, like Fisher. For alternative criteria see Walsh [1901], Fisher [1922], Frisch [1930], Samuleson & Swamy [1974], Eich-

horn & Voeller [1976], Diewert [1993], and Balk [1995]. The whole idea, I will say, is rather confusing for the moment, and its theorists seems not to have a clear picture about axioms, derived propositions, and operational results. But it might be the case that index theory does not have to obey and subordinate to fundamental results in the mathematical branches of real analysis and algebra.

The approach is not straightforward and clear. First, a number of people claim that they have shown the approach to be inconsistent, as Frisch [1930], Wald [1937], Swamy [1965], Samuleson & Swamy [1974], and Eichhorn & Voeller [1976], i.e. that such an index does not exist, which I consider a triviality under the operation rules for vectors. Swamy [1965] and Eichhorn & Voeller [1976] mean that the inconsistency theorems of Walsh [1901] and Frisch [1930] are not valid. And Eichhorn & Voeller [1976] claim that they provide the complete solution. Second, the index theorists do not yet agree which criteria should count. Swamy [1965] means that the issue has to be settled within the economic framework. But, I belief the whole idea is dead. Third, it seems that the approach is a bastard application of the Abelian group, the axiom of real analysis, and the operation rules of vectors. Samuelson & Swamy [1974] mean that desirable properties for an index of many items have been postulated by analogy with the case of the price of a single good. Fourth, the approach does not yield desired data for model applications. In their determination to find the ultimate index, they have forgotten the imperative character of the pragmatic criteria.

For the moment I just emphasize that the axiomatic approach is of minor interest when determine a proper index form for analytic model application. In fact, one should not consider such kind of criteria at all. One main reason is that you will be twisted in wrong direction.

3.4 The classical indices and some of its properties

A nice algebraic connection can be notified between the index format of Laspeyres, Paasche, and Bowley-Fisher[6]. From the value index

$$(12) \qquad V_t = \frac{p_t q_t}{p_o q_o}$$

i.e. the change in current value between two periods, we can by simple algebraic expansion obtain the following exhaustive expressions,

$$(13) \quad \frac{p_t q_t}{p_o q_o} = \frac{p_o q_t}{p_o q_t} \cdot \frac{p_t q_t}{p_o q_o} = \frac{p_t q_t}{p_o q_t} \cdot \frac{p_o q_t}{p_o q_o} = P_P \cdot Q_L$$

$$(14) \quad \frac{p_t q_t}{p_o q_o} = \frac{p_t q_o}{p_t q_o} \cdot \frac{p_t q_t}{p_o q_o} = \frac{p_t q_t}{p_t q_o} \cdot \frac{p_t q_o}{p_o q_o} = Q_P \cdot P_L$$

[6] Fisher acknowledged in his writing the formula of Bowley. Nevertheless it is more known as the Fisher's ideal index, or just Fisher's index. He called it the King of indices. The contribution of Bowley is unfairly unknown today. I like to appraise his

$$(15) \quad \frac{p_t q_t}{p_o q_o} = \sqrt{\left(\frac{p_t q_t}{p_o q_o}\right)^2 \cdot \frac{p_o q_t}{p_o q_t} \cdot \frac{p_t q_o}{p_t q_o}} =$$

$$= \sqrt{P_P \cdot P_L}\sqrt{Q_P \cdot Q_L} = P_{B,Fi} \cdot Q_{B.Fi}$$

Thus, in pragmatic sense there is a workable mathematical relation of exhausting nature under the value index between the Laspeyres, Paasche, and Bowley-Fisher formulas in such a way that the value index can be expressed as the product of a weighted price term and a weighted quantity term of the same index structure.

Immediate we get the expressions in fixed prices

$$(13') \quad \frac{p_t q_t}{P_P} = p_o q_t \qquad \textit{(Laspeyres quantity)}$$

$$(14') \quad \frac{p_t q_t}{P_L} = p_o q_o \cdot Q_P \qquad \textit{(Paasche quantity)}$$

$$(15') \quad \frac{p_t q_t}{P_{B,Fi}} = p_o q_o \cdot Q_{B.Fi} \qquad \textit{(Bowley-Fisher quantity)}$$

Analogously we get the corresponding three expressions of prices connected to the current prices, like

$$(13'') \quad \frac{p_t q_t}{Q_L} = p_o q_o \cdot P_P \qquad \textit{(Paasche price)}$$

$$(14'') \quad \frac{p_t q_t}{Q_P} = p_t q_o \qquad \textit{(Laspeyres price)}$$

$$(15'') \quad \frac{p_t q_t}{Q_{B.Fi}} = p_o q_o \cdot P_{B,Fi} \qquad \textit{(Bowley-Fisher price)}$$

Thus, only Laspeyres quantities and Laspeyres prices are additive, as

$$\frac{p_t q_t}{P_P} = p_o q_t = p_o^1 q_t^1 + p_o^2 q_t^2 + \ldots + p_o^n q_t^n$$

$$\frac{p_t q_t}{Q_P} = p_t q_o = p_t^1 q_o^1 + p_t^2 q_o^2 + \ldots + p_t^n q_o^{n1}$$

Such decomposition is not possible for the other classical indices.

The additive property is also very crucial when you come to national accounting in fixed prices. For example you can restructure the industries, or the sectors without changing the aggregate GNP in fixed prices, which in model application is an important and useful property. Thus, regarding national account compilation, we have to restrict our efforts to (13), and nothing else in the case of a fixed basket. The additive property is also one of the desirable properties we should be careful looking for, especially for improving the possibilities of straight forward and fruitful model application. In other words, the additive property is

This shows what kind of index structure we are looking for. The issue is discussed in Hansson [1996][1998], where it is stated that under the fixed basket approach you have to do a choice. If you are interested in a pure price analyses, then you should use expression (14), while if you are interested in a pure quantity measurement and analyses, like for example the accounting and use of National Accounts in fixed prices, then you should use (13). The reasons for this, as we can see, are pure mathematical.

The main reason is that Paasche price index is quantity biased in the sense that the index will show a price change for any quantity change even if there are no changes in existing prices. And analogously, that the Paasche quantity index is price bias in the sense that the index will show a quantity change for any price changes despite if there are no changes in existing quantities.

It can be notated that expression (13), because that Laspeyres quantities are additive, which is a crucial property for national accounting, also is the international recommendation by United Nations; though in the original version of SNA 1993 the Fisher version of (15) was the choice, probably they were blended at the time by the ultimate chase of the superlative index, and of Fisher's King of Indices. A line I consider as badly misleading when you are looking for good possibilities to maintain accurate compilations for acceptable model applications.

In a world were there are no changes in the number of different and unique goods and services, (13) and (14) is the best we can do, as there will not be any better structures of index. But in a dynamic world where the numbers goods and services and there qualities continuously are changing, these formulas will not do. In that kind of world we have to look for an extended version of the Laspeyres formula structure. In Section 4, I will discuss the reasons for such an option and why we have to look for such a solution. But first a deep relation among the classical indices.

3.5 A deep and beautiful relation

Bortkiewicz [1922][1924] found a nice and exhaustive relation between the indices of Laspeyres and Paasche, which Allen [1975] named statistical relations[7]. Here I will chose an alternative, but more simpler and straightforward derivation.

We can rewrite (12) like

$$(16)\quad V_t = \frac{p_t q_t}{p_o q_o} = \frac{\sum_i p_t^i q_t^i}{p_o q_o} = \sum_i \frac{p_o^i q_o^i}{p_o q_o} \cdot \frac{p_t^i q_t^i}{p_o^i q_o^i} = \sum_i w_{oo}^i V_t^i$$

From (13) and (14) we also have

$$(17)\quad V_t = P_P Q_L = P_L Q_P$$

Dividing by $P_L Q_L$ gives

$$(18)\quad \frac{V_t}{P_L \cdot Q_L} = \frac{P_P}{P_L} = \frac{Q_P}{Q_L} = B_H$$

Set $w_{oo}^i = p_t^i q_t^i / p_o q_o$; $x_t^i = p_t^i / p_o^i$; and $z_t^i = q_t^i / q_o^i$

Then we have that

$$(19)\quad V_t = \sum_i w_{oo}^i V_t^i = \sum_i w_{oo}^i x_t^i z_t^i = \mathrm{E(XZ)},$$

$$(20)\quad P_L = \sum_i w_{oo}^i x_t^i = \mathrm{E(X)},\ Q_L = \sum_i w_{oo}^i z_t^i = \mathrm{E(Z)}$$

Relation (18) can now easily be rewritten like

$$(21)\quad B_H = \frac{Q_P}{Q_L} = \frac{P_P}{P_L} = \frac{V_t}{Q_L \cdot Q_P} = \frac{\mathrm{E(XZ)}}{\mathrm{E(X)} \cdot \mathrm{E(Z)}} =$$

$$= 1 + \frac{\mathrm{Cov(XZ)}}{\mathrm{E(X)} \cdot \mathrm{E(Z)}} = 1 + r_{XZ} \frac{s_X \cdot s_Z}{\mathrm{E(X)} \cdot \mathrm{E(Z)}}$$

where r_{XZ} is the coefficient of correlation between X and Z. Now following inequalities are given

$$P_L \geq P_{FB} \geq P_P \qquad Q_L \geq Q_{FB} \geq Q_P \quad \text{when} \quad -1 \leq r_{XZ} \leq 0$$

$$P_L \leq P_{FB} \leq P_P \qquad Q_L \leq Q_{FB} \leq Q_P \quad \text{when} \quad 1 \geq r_{XZ} \geq 0$$

Equalities hold when

$$r_{XZ} = 0,$$

i.e. when $\mathrm{Cov(XZ)} = \mathrm{E(XZ)} - \mathrm{E(X)} \cdot \mathrm{E(Z)} = V_t - P_L \cdot Q_L = 0$,

$$\text{or when } p_t^i = dp_o^i, \quad \text{or when } q_t^i = mq_o^i,$$

as than $s_X^2 = \mathrm{E}(X^2) - \mathrm{E(X)} \cdot \mathrm{E(X)} = d^2 - d^2 = 0$,

$$s_Z^2 = \mathrm{E}(Z^2) - \mathrm{E(Z)} \cdot \mathrm{E(Z)} = h^2 - h^2 = 0.$$

Thus, the relation found by the young von Bortkiewicz in the 1920s and published in an international unknown Swedish journal, more interested in odd internal subjects, is of great interest. But we have to be careful when interpret it. Allen's economic interpretation[8] I belief is misleading. When $r_{XZ} > 0$, Allen mean that market is supply dominated, and when $r_{XZ} < 0$, than the market is demand dominated. This I belief is nonsense in a pragmatic situation, especially when you come to national accounting and price measurement for the CPI. If the suppliers manage to set the price, nothing will say that traded quantities thereby will increase. Further, if the demanders manage to set a lower price, there is nothing about that the traded thereby will increase in reality. Moreover there are no ways to show such hypothesis empirically. Furthermore, we have to take in to account that Paasche price index is

[7] About Bortkiewicz's derivation see Allen [1975, p. 63].

[8] Allen [1975, p. 64].

quantity biased, and that Paasche quantity index is price biased. And what about the theory that prices and quantities tend to increase in the upward slope of the business cycles and decrease in its downward slope.

For the moment I will regard (20) as strict mathematical relation, or identity, by which you can compile the missing index. For example if you have a Paasche price index, you can by (20) compile the price index by Laspeyres, and vice versa. But it might be recommended to the compilations directly by a proper statistical sample.

4 A pragmatic approach

Since the 1950s there is a continuous claim for accurate and meaningful economic statistics and national accounts. All the time there have been complaints that our figure cannot be used in model applications, such as economic analyses, formatting economic policy, forecasting, econometric modeling, and testing economic hypotheses. International there is a huge literature in these matters, and in Sweden the complaints and quality requirements mostly have been seen in the press. Rather detailed information about this discussion can be seen in Hansson [2001]. See also Hansson [2000a]. And recently *The Commission on the Review of Economic Statistics*, SOU 2001, concluded after asking a number of main users that there was four main areas where there are especially urgent need to develop new statistics namely price measurements for the business service sector, IT-statistics, new statistics for comprehensive input-output table, and measures of capital stock. Those were the areas most frequently brought up during the interviews with the users. They are also the areas the Commission mean, that need to be improved above all, in order to analyze productivity growth in a more accurate way and the impact of the changes in the economy which is usually refer to the New Economy.

Obviously there are problems with our present economic statistics for the users. And the question is what we can do about it. In Hansson [2001] I argue that nothing can be done within the conventional approach, and urge that we have to seek an alternative agenda. In the following I will discuss such an approach, likely called the pragmatic approach. Preferable it could also be named the analytic approach, as it focuses upon such a mathematical philosophy, which is a basic approach for meaningful model application.

4.1 Pragmatic criteria

Data criteria and data properties in model application, I imagine, must coincide with the subject matter of the analysis, i.e the restrictions and requirements determined by the factual flows and stocks of utilities and financial items, the interdependency between economic variables, the mathematical restrictions, and the possibilities econometric theory opens. These kind of criteria are pure analytical. None of the above indices satisfy such restrictions and requirements. An alternative approach has to be formulated in a proper and pure mathematical fashion. Debreu utility, characteristics, time and location, are not enough. At a beginning, at least six criteria have to be considered.

1. Value consistency, i.e.

$$p_t^i q_t^i,\ p_o^i q_t^i,\ p_t^i q_o^i,\ p_o^i q_o^i,$$

where *i* is utility *i*, and t time of trade or transaction. None of the so called superlative indices meet this criterion.

2. Transaction and space consistency, i.e.

$$p_t^{is} q_t^{is},\ p_o^{is} q_t^{is},\ p_t^{is} q_o^{is},\ p_o^{is} q_o^{is},$$

where *s* is the simultaneous double accounting for agents involved. Involved agents can be located at different locations, for example, a fruit trader in Borlänge buys his fruit from Italy, and the grocery store's owner and employees and customers can live in different regions. For the superlative indices to meet this criterion and the following, a far too complicated compilation is needed.

3. Time consistency, i.e.

$$\sum_{t\in T} p_t^{is} q_t^{is},\ \sum_{t\in T} p_o^{is} q_t^{is},\ \sum_{t\in T} p_t^{is} q_o^{is},\ \sum_{t\in T} p_o^{is} q_o^{is}$$

where T is a certain period of time such as day, week, month, quarter and year. Normally it is enough to set t = day. By cumulative addition over t, we then obtain time consistency over week, month, quarter and year consecutively. The conventional way, for example in CPI, to collect prices just for certain day, say the midday of the month, will never give time consistency. For that you have to compile price and quantity indices for every day the goods and services are traded.

4. Neutrality

The measures must be pure, i.e. price measures are not affected by quantity changes, and quantity measures are not affected by price changes. Paasche indices are not neutral in this sense, while Laspeyres indices are, see Hansson [1996] for a discussion. None of the superlative indices meet this criterion.

5. Unbiased

Observed price and quantities should be the true ones. For a static basket, this property is a triviality. But for a dynamic basket, some decisions must be considered. None of the indices above meet this criterion in construction.

6. Exhaustive

This is the possibility to factorise the traded value entirely into the true price and quantity factors. One implication of this criteria, is that the factorised price and quantity indexes are of the same form, like for example in (13) and (14). None of the superlative indices meet this criterion in construction.

None of published indices, economic statistics, or national accounts meet any of these criterion. In addition, no price and quantity index under the value index simultaneously fulfils all these criteria. An implicit proof for that is shown by (13') - (15'). As we can be seen, the Laspeyres quantity and price indices are left to be considered.

Under the fixed basket approach, quantities are additive under fixed prices, but not prices, formula (13'); and prices are additive under fixed quantities, but not quantities, formula (4. And the indices of (15') have to be left. None of them is additive. Thus, you cannot simultaneously, under the value index, study pure price and pure quantity changes. It must be done separately. And, most of all you cannot study a dynamic economy carefully or accurately. For that some extension has to be considered. That is discussed is discussed rather extensively in Hansson [1996].

4.2 Operational index structure

If we must maintain value consistency in a dynamic situation, then the following valuations, or taxations, is a possible and workable approach in operational compilation that can be considered,

Actual trade in period t: $\sum_i p_t^i q_t^i = p_t q_t$;

Fixed prices: $\sum_i \hat{p}_o^i q_t^i = \hat{p}_o q_t$, $\sum_j p_o^j \hat{q}_t^j = p_o \hat{q}_t$

Fixed quantities: $\sum_i p_t^i \hat{q}_o^i = p_t \hat{q}_o$, $\sum_j \hat{p}_t^j q_o^j = \hat{p}_t q_o$

Actual trade in period o $\sum_j p_o^j q_o^j = p_o q_o$

where the symbol "^" indicates the prices and quantities that are missing, and must be implemented. By apply such an extension it is possibly in a way to construct index forms with almost authentic weights that almost stand all the pragmatic criteria. The only crucial point is the determination of the missing values. Hansson [1996][1998] discusses extensively how to estimate these values. To what extent, or in what sense, is it now possible to formulate price and quantity indices satisfying the pragmatic criteria? One way is to impute the corresponding values compiled within the internal book keeping system of the firms.

To simplify matters and pin point the approach, assume that utilities 1 and 2 are traded in period o, and utility 3 in period t, as in

(15) $$V = \frac{p_t^3 q_t^3}{p_o^1 q_o^1 + p_o^2 q_o^2}$$

Algebraic expansion of V by missing prices as weights gives

(16') $$V = \frac{p_t^3 q_t^3}{\hat{p}_o^3 q_t^3} \cdot \frac{\hat{p}_o^3 q_t^3}{p_o^1 q_o^1 + p_o^2 q_o^2} = \hat{P}_{Pp_o} \cdot \hat{Q}_{Lp_o}$$

(16'') $$V = \frac{p_t^3 q_t^3}{\hat{p}_t^1 q_o^1 + \hat{p}_t^2 q_o^2} \cdot \frac{\hat{p}_t^1 q_o^1 + \hat{p}_t^2 q_o^2}{p_o^1 q_o^1 + p_o^2 q_o^2} = \hat{Q}_{Pp_t} \cdot \hat{P}_{Lp_t}$$

From (16') we see that the quantity index of Laspeyres type, Q_{Lpo}, meets all the pragmatic criteria, especially that its fixed prices are additive. One can discuss to what extent it is neutral, since the outcome depends upon the valuation of the missing prices of utility 3. But that can be handled carefully, since we can investigate different outcomes. Further, despite the degree of bias, it must be regarded as an accurate quantity measure. We probably cannot do better. The price index of Paasche, P_{Ppo}, does measure the price change of the traded utilities. But it is not neutral. The important question, though, is whether or not it is acceptable as a price index. The Paasche price index is the proper counterpart to the Laspeyres quantity index. And if a price index shall contain just traded utilities in the current period, which seems to be a correct answer, this is the best we can do in a dynamic situation.

By using estimates of missing quantities as weights, we obtain

(17') $$V = \frac{p_t^3 q_t^3}{p_o^1 \hat{q}_t^1 + p_o^2 \hat{q}_t^2} \cdot \frac{p_o^1 \hat{q}_t^1 + p_o^2 \hat{q}_t^2}{p_o^1 q_o^1 + p_o^2 q_o^2} = \hat{P}_{Pq_t} \cdot \hat{Q}_{Lq_t}$$

(17'') $$V = \frac{p_t^3 q_t^3}{p_t^3 \hat{q}_o^3} \cdot \frac{p_t^3 \hat{q}_o^3}{p_o^1 q_o^1 + p_o^2 q_o^2} = \hat{Q}_{Pq_o} \cdot \hat{P}_{Lq_o}$$

The quantity indices are misleading. The Laspeyres price of (17') is a measure of the change in outlays expressed in the base period quantities. But can that be a true measure of the compound change in prices in traded utilities? Regarding price changes, shall such a measure involve traded utilities in either periods, or just traded utilities in the current period? Anyway, its counterpart, the Paasche quantity index, yields a totally wrong picture of the quantity change. This does not seem to be useful for model applications. An interesting question is whether a case with a large number of traded utilities in both periods shall be regarded as an acceptable approximation.

Thus, through (16') we have an acceptable measure for the compound Laspeyres quantity in a dynamic economy. Also, the Paasche price index can be regarded as an acceptable measure of the compound price change considering current traded utilities, though it is not neutral. We must accept that, as an entirely neutral measure does not seem to be obtainable in a dynamic situation.

The weak point is $\hat{p}_o^3$, but in a sense the strong point; strong in the sense that we are in full control of it. Note that $\hat{p}_o^3$ here is our weight item, and our main problem is to find an acceptable weight structure. We here know exactly what weight value we have imposed on the measure. And we can entirely inquire its importance and qualities, as we all the time know what we are doing and are in possession of all the needed information for that. In conventional statistical price estimates, this is not the case. Particular in a highly dynamic situation, in which we very quickly lose control over our basket to be measured.

You can always analyse different outcomes entirely. This is far from the case for superlative indices, or hedonic indices or published figures. With our present statistical concept, it is not possible to measure our economy properly in current and fixed prices. For the moment, there are signs of a heavy underestimation and misspecification of the cyclical path of our economy.

4.3 Some isolated factual outcomes

The structure of (15) is chosen to pinpoint the basic properties of the approach. In a real situation is does not look that blunt way, especially when you use the formula in a daily fixed price compilation. We can look at two situations to show that the bias imposed by $\hat{p}_o^3$ probably in a factual situation be can ignored in statistical sense. From one day to the other there either no change in the traded structure of services and their prices, or to some extent a change. This change might be very slight and ignorable between two consecutive days, but over a month, or a quarter or a year it might be rather considerable, but still a core from the chosen base day might be rather dominating. Which mean though that we all the time must watch the change very carefully.

Say we have no change, than we can image the following situation.

$$(18) \qquad V = \frac{p_t^1 q_t^1 + p_t^2 q_t^2 + p_t^3 q_t^3}{p_o^1 q_o^1 + p_o^2 q_o^2 + p_o^3 q_t^3}$$

Algebraic expansion of V by missing prices as weights gives

(18')

$$V = \frac{p_t^1 q_t^1 + p_t^2 q_t^2 + p_t^3 q_t^3}{p_o^1 q_t^1 + p_o^2 q_t^2 + p_o^3 q_t^3} \cdot \frac{p_o^1 q_o^1 + p_o^2 q_o^2 + p_o^3 q_t^3}{p_o^1 q_o^1 + p_o^2 q_o^2 + p_o^3 q_o^3} = P_{Pp_o} \cdot Q_{Lp_o}$$

(18'')

$$V = \frac{p_t^1 q_t^1 + p_t^2 q_t^2 + p_t^3 q_t^3}{p_t^1 q_o^1 + p_t^2 q_o^2 + p_t^3 q_o^3} \cdot \frac{p_t^1 q_o^1 + p_t^2 q_o^2 + p_t^3 q_o^3}{p_o^1 q_o^1 + p_o^2 q_o^2 + \hat{p}_o^3 q_t^3} = Q_{Pp_t} \cdot P_{Lp_t}$$

As all three services are traded in both periods there is no need of any price estimation, and formula work like the classical situation with a fixed basket of services. And we see there are no problems involved when perform a pure quantity analyses through (18') and a pure price analyses through (18''). Between two consecutive days, it more or less will look like these index structures.

Say instead we have a slight change in period *t*; like that Service 2 traded in period *o* is not traded in period t, while Service 3 not traded in period *o* is traded in period *t*, than we have the following situation.

$$(19) \qquad V = \frac{p_t^1 q_t^1 + p_t^3 q_t^3}{p_o^1 q_o^1 + p_o^2 q_o^2}$$

Algebraic expansion of *V* by missing prices gives

$$(19') \qquad V = \frac{p_t^1 q_t^1 + p_t^3 q_t^3}{p_o^1 q_t^1 + \hat{p}_o^3 q_t^3} \cdot \frac{p_o^1 q_o^1 + \hat{p}_o^3 q_t^3}{p_o^1 q_o^1 + p_o^2 q_o^2} = \hat{P}_{Pp_o} \cdot Q_{Lp_o}$$

$$(19'') \qquad V = \frac{p_t^1 q_t^1 + p_t^3 q_t^3}{\hat{p}_t^1 q_o^1 + \hat{p}_t^2 q_o^2} \cdot \frac{\hat{p}_t^1 q_o^1 + \hat{p}_t^2 q_o^2}{p_o^1 q_o^1 + p_o^2 q_o^2} = Q_{Pp_t} \cdot \hat{P}_{Lp_t}$$

In (19') we see that the Laspeyres quantity Q_{Lpo}, is a true measure of the quantity change, while its counterpart P_{Ppo}, the Paasche price index, depict the price change of traded services. So, (19') is okay for pure quantity analysis, and acceptable in a price analysis. In (19'') there are some problems. Hardly we can use it for a pure analysis, as the Laspeyres' price term just involves these services that was traded in period *o*, and whether or not they are traded in period *t*.

By using estimates of missing quantities as weights, we have

$$(19alt') \qquad V = \frac{p_t^1 q_t^1 + p_t^3 q_t^3}{p_o^1 \bar{q}_t^1 + p_o^2 \bar{q}_t^2} \cdot \frac{p_o^1 \bar{q}_t^1 + p_o^2 \bar{q}_t^2}{p_o^1 q_o^1 + p_o^2 q_o^2} = \bar{P}_{Pq} \cdot \bar{Q}_{Lq}$$

$$(19alt'') \qquad V = \frac{p_t^1 q_t^1 + p_t^3 q_t^3}{p_t^1 q_o^1 + p_t^3 \bar{q}_o^3} \cdot \frac{p_t^1 q_o^1 + p_t^3 \bar{q}_o^3}{p_o^1 q_o^1 + p_o^2 q_o^2} = \bar{Q}_{Pq_o} \cdot \bar{P}_{Lq_o}$$

Neither of these expressions can be used in a quantity analysis, as no new traded services are involved quantity indices. None of the price terms can be regarded as a measurement of a price change. First of all they are measurements of the turnover change; in (19alt') expressed in current quantities, and in (19alt'') expressed in fixed quantities. Obviously these price indices can give an entirely wrong image of the price change.

Assume now a situation were we have a Service 3 traded in period *t*, but not traded in period *o*, than we have a structure like

$$(20) \qquad V = \frac{p_t^1 q_t^1 + p_t^2 q_t^2 + p_t^3 q_t^3}{p_o^1 q_o^1 + p_o^2 q_o^2}$$

Algebraic expansion of *V* by missing prices as weights gives

(20')

$$V = \frac{p_t^1 q_t^1 + p_t^2 q_t^2 + p_t^3 q_t^3}{p_o^1 q_t^1 + p_o^2 q_t^2 + \hat{p}_o^3 q_t^3} \cdot \frac{p_o^1 q_t^1 + p_o^2 q_t^2 + \hat{p}_o^3 q_t^3}{p_o^1 q_o^1 + p_o^2 q_o^2} = \hat{P}_{Pp_o} \cdot \hat{Q}_{Lp_o}$$

(20'')

$$V = \frac{p_t^1 q_t^1 + p_t^2 q_t^2 + p_t^3 q_t^3}{p_t^1 q_o^1 + p_t^2 q_o^2} \cdot \frac{p_t^1 q_o^1 + p_t^2 q_o^2}{p_o^1 q_o^1 + p_o^2 q_o^2} = Q_{Pp_t} \cdot P_{Lp_t}$$

And by using estimates of missing quantities, we have

$$(20alt') \qquad V = \frac{p_t^1 q_t^1 + p_t^2 q_t^2 + p_t^3 q_t^3}{p_o^1 q_t^1 + p_o^2 q_t^2} \cdot \frac{p_o^1 q_t^1 + p_o^2 q_t^2}{p_o^1 q_o^1 + p_o^2 q_o^2} = P_{Pq} Q_{Lq}$$

(20alt'')

$$V = \frac{p_t^1 q_t^1 + p_t^2 q_t^2 + p_t^3 q_t^3}{p_t^1 q_o^1 + p_t^2 q_o^2 + p_t^3 \bar{q}_o^3} \cdot \frac{p_t^1 q_o^1 + p_t^2 q_o^2 + p_t^3 \bar{q}_o^3}{p_o^1 q_o^1 + p_o^2 q_o^2} = \bar{Q}_{Pq_o} \cdot \bar{P}_{Lq_o}$$

Again, only (20') make sense.

Suppose Service 3 traded in period *o*, is not traded in period *t*, than instead we have the following structure

(21) $$V = \frac{p_t^1 q_t^1 + p_t^2 q_t^2}{p_o^1 q_o^1 + p_o^2 q_o^2 + p_o^3 q_o^3}$$

Algebraic expansion of V by missing prices as weights gives

(21')
$$V = \frac{p_t^1 q_t^1 + p_t^2 q_t^2}{p_o^1 q_t^1 + p_o^2 q_t^2} \cdot \frac{p_o^1 q_o^1 + p_o^2 q_t^2}{p_o^1 q_o^1 + p_o^2 q_o^2 + p_o^3 q_o^3} = P_{Pp_o} \cdot Q_{Lp_o}$$
(21'')
$$V = \frac{p_t^1 q_t^1 + p_t^2 q_t^2}{p_t^1 q_o^1 + p_t^2 q_o^2 + \hat{p}_t^3 q_o^3} \cdot \frac{p_t^1 q_o^1 + p_t^2 q_o^2 + \hat{p}_t^3 q_o^3}{p_o^1 q_o^1 + p_o^2 q_o^2 + p_o^3 q_o^3} = \hat{Q}_{Pp_t} \cdot \hat{P}_{Lp_t}$$

Observe that in (21') no imputations of price weights are needed.

And algebraic expansion by missing quantities as weights will here give the same result, as

(21alt')
$$V = \frac{p_t^1 q_t^1 + p_t^2 q_t^2}{p_t^1 q_o^1 + p_t^2 q_o^2} \cdot \frac{p_t^1 q_o^1 + p_t^2 q_o^2}{p_o^1 q_o^1 + p_o^2 q_o^2 + p_o^3 q_o^3} = P_{Pq_o} \cdot Q_{Lq_o}$$
(21alt'')
$$V = \frac{p_t^1 q_t^1 + p_t^2 q_t^2 + p_t^3 q_t^3}{p_t^1 q_o^1 + p_t^2 q_o^2} \cdot \frac{p_t^1 q_o^1 + p_t^2 q_o^2}{p_o^1 q_o^1 + p_o^2 q_o^2} = Q_{Pq} \cdot P_{Lq}$$

To be sure, only pure quantity analyses seem to be the proper thing, and we must except its biased counterpart in a dynamic price analyse. So analogously the expressions (16'), (18'), (19'), (20'), and (21') must be considered.

5 The empirical experiment

Formula (16'), and its extended versions of (18'), (19'), (20'), and (21'), has shown, by experience, to be extremely effective when instantly compiling any quality changes and extremely dynamic situations where the traded utility mix is constantly changing, like in the freight business. It is effective in the daily taxation of current and fixed prices within the firm. These daily figures than are consecutively added to monthly figures, quarterly figures and yearly figures. In Diagram 6 the yearly figures are shown. And in Diagram 7 the corresponding figures are shown when using our present method of fixed prise compilation. As can be seen, the outcomes are completely contrary. The prices in Diagram 7a are considerable down sloping, while in Diagram 6a the factor price index used in our published figures are considerable upward sloping. In Diagram 7b the quantities show rather nice looking part of a upward sloping part of a business cycle, while corresponding quantities in Diagram 7b show up a more divide and contradictory features.

If the authentic figures of freight transportation on lorries look are in line with Diagram /, than our published figures surely have to be looked over and revised.

6 About the experiment

In Sweden there are no index officially published about the payments for freight transports. But Hansson [1996] [2001] describes an ongoing experiment after another view, quite different from the above mentioned. Using the equations of (16') a number of efficient features can be mentioned:

1. Daily current and fixed prices is compiled for the clients entirely payment, and its entire split between the forwarding agent and contracted haulage contractors.

2. It is not a sample of representative, services, or some kind of model pricing, nor some kind of hedonic compilation, which is some of the methods so far used internationally. It is a measurement of the entirely payment of the clients. It is an entirely measurement of the split of the payment between the forwarding agent and the used haulage contractors.

3. Due to the daily compilation in current and fixed prices, we accurate figures for daily, weekly, monthly, quarterly, and yearly changes in current and fixed prices. Conventional published figures have not that property. For example, CPI can only depict monthly change within in interval of at least ± 2 %. Yearly change might be rather accurate.

4. The compilation in fixed and current prices is done within in the forwarding agents book keeping system in the moment the clients are taxed for the freight transportation accepted. If the clients accept the required payments, and if the haulage contractors are with their share, and if the forwarding agents are satisfied with the whole thing, then the compiled figures in current and fixed prices cannot be more accurate. This mean that we don't have to organize some more or less doubtful and costly editing of delivered data. The cli
ents, the forwarding agents, and the haulage contractors in their ordinary and professional business have already done the editing for the statistic bureau.

5. The statistical bureau do not have to send out hundreds of questionnaires to haulage contractors, as the forwarding agent already have the wanted information within is internal book keeping system for direct access.

6. The compiled figures for the clients, the forwarding agents, and the haulage contractors are consistent in the sense that they entirely are connected to each other, as the figures simultaneously are compiled within the forwarding agents book keeping system. This is a crucial feature for model application, forecasting, deep and meaningful economic analyses that do not exist in our conventional compilation for the national accounts. In conventional statistical compilation, these figures are gathered through different and independent statistical production.

7. The sample is not separately over the clients, forwarding agents, and haulage contractors. It is just over forwarding agents, as they are in the possession of all required information through its book keeping system.

7. All information required is transmitted in a week after the end of the month. Even in two days have turned out a reality. They're a number of important and efficient implications of this fastness in reporting, and its authentic kind of accurateness. In a week or fortnight we are in the possession to publish accurate and time consistent economic. And most important, we are in the possession to publish time consistent and accurate monthly accounts in a fortnight after the ends of the month. The same is true for the publication of quarterly as well as yearly national accounts, which are all time consistent and accurate in the sense of authentic national accounts. This means that we can publish timely and burning national accounts all the time, which our users constantly are demanding, especially the current business analyzers. We do not have to publish a definite and doubtful totally inaccurate version two years after the end of the year, neither a number of preliminary versions in between. About that see Hansson [1998a][200a][2001].

8. The weak point is the imputed price $\hat{p}$. Is it an accurate approach, and how should $\hat{p}$ be estimated. Hansson [1996][1998b][2000b] has a discussion about that. In these papers I point out that the problem we have to solve is to find a proper and acceptable weight. In this sense the imputation of $\hat{p}$ is acceptable, and to my concern the best we can do in a dynamic situation. The second question to answer is what kind of estimate we shall use. I argue that the most proper estimate is to be found within the forwarding agents accounting system, such a nearby index like for nearby clients, or nearby market, or nearby transport. If there is a tendency of decreeing pricing it is good thing to follow that tendency. If there is actual price list, it is suggestible to pick up the imputed price from that list. The good point, what so ever imputation policy chosen, we can entirely investigate a number of more or less reasonable alternatives, and see to what extent it matters.

References

Allen, R. G. D., [1975], "*Index Numbers in Theory and Practice*," MacMillan Press Ltd, Edingburgh, 1975.

Balk, B.M, [1995], "Axiomatic Price Index Theory: A Survey," *International Statistical Review, 63*, p 69-93.

Bortkiewicz, L. von, [1922], "Zweck und Struktur einer Preisindexzahl", *Nordisk Statistisk Tidskrift*, 1, 1922.

Bortkiewicz, L. von, [1924], "Zweck und Struktur einer Preisindexzahl", *Nordisk Statistisk Tidskrift*, 3, 1924.

Bowley, A. L. [1899], „Wages, Nominal and Real", In *Dictionary of Political Economy*, Vol 3, R.H.I. Palgrave (ed), London, Macmillan, pp 640-651.

Diewert, W.E, [1993], "The Early History of Price Index Research," *Essays in Index Numbers, Volume I*, Eds. W.E: Diewert and A.O. Nakamura.

Edgeworth, F., Y., [1888], "Some New Methods of Measuring Variation in General Prices", *Journal of the Royal Statistical Society* 51, pp 346-368. Also in *Papers Relating to Political Economy*, Vo 1, New York, Burt Franklin, 1925.

Eurostat, [2001], "*Handbook on Price and Volume Measures in National Accounts, Further Discussions.*" Eurostat B1/CN 471 e, 2001.

Eichhorn, W. & J. Voeller, [1976], "Theory of the Price Index, Fisher's Test Approach and Generalizations," Lectures Notes in Economics and Mathematical Systems, Vol. 140, Springer-Verlag, Berlin.

Ferger, W.F. [1946], "Historical Note on the Purchasing Power Concept and Index Numbers", *Journal of the American Statistical Association* 41, 1946, pp53-47.

Fisher, I, [1911], *The Purchasing Power of Money*, Macmillan, London, 1911.

Fisher, I, [1922], *The Making of Index Numbers*, Boston, 1922.

Frisch, R, [1930], "Necessary and Sufficient Conditions Regarding the Form of an Index Number which Shall Meet Certain of Fisher's Tests," *Journal of the American Statistical Association, XXV*, March, p 397-406.

Hansson, Karl-Gustav [1990], "Bärkraftighetsanalys och felskattning utifrån de svenska nationalräkenskaperna med avseende på den svenska ekonomin. Förutsättninar och två enkla felskattningsanalyser avseende lastbilsåkerier och godstransporter på landsväg. ", Unpublished study, 62 pp, internally circulated at *Department of National Accounts, Statistics Sweden.*

Hansson, Karl-Gustav [1996], "Indexproblem vid en övergång till heltäckande on-line rapportering i Nationalräkenskaperna", Arbetspapper nr 5, November 1996, *Konjunkturinstitutet*. Summary in English.

Hansson, Karl-Gustav [1998], Price and Quantity Indices when New and Vanishing Products are Considered. The Classical Indices Reconsidered. i *Official Statistics in a Changing World*, Third conference on Methodological Issues in Official Statistics, Stockholm, October 12-13 1998.

Hansson, Karl-Gustav [2000a], "Totally Consistent Monthly and Yearly National Accounts". *ICES-II Proceedings of the Second International Conference on Establishment Surveys*, American Statistical Association, Alexandria Virginia 2000.

Hansson, Karl-Gustav [2000b], "Entirely Consistent and Operational Price and Quantity Indices". *Statistics and Democracy*. Proceedings of the 4th international Conference on Methodological Issues in Official Statistics, Stockholm October 12-13 2000.

Hansson, Karl-Gustav [2001], "Improved National Accounts", Paper presented at *Stockholm Quality Conference*, May 14-15, 2001.

Jevons, W., S., [1865], "Variations of Prices and the Value of Currency since 1762", *Journal of the Royal Statistical Society*, 28 June 1865, pp 294-325.

Jevons, W., S., [1884], "*Investigations in Currency and Finance*", Macmillan, London 1884.

Keynes, J. Maynard, [1930], "*Treatise of Money*", Vol 1, Macmillan, London 1930.

Lowe, Joseph, [1823], "*The Present State of England in Regard to Agriculture, Trade, and Finance*" second edition, London, Longman, Hurst, Rees, Orme and Brown, 1823.

Marshall, Alfred, [1887], "Remedies for Fluctuations of General Prices", *Contemporary Review* 51, 1887, Reprinted in *Memories of Alfred Marshall*, A. C. Pigou (ed), London, Macmillan, 1925.

Pigou, A.C. [1912], "*Wealth and Welfare*", London, Macmillan, 1912.

Samuelson, P.A, & S. Swamy, [1974], "Invariant Economic and Canonical Duality: Survey and Synthesis," *American Economic Review, Vol. 64 No 4*, p 566-593.

Sidgwich, H., [1883], *"The Principles of Political Economy"*, London, MacMillan, 1883.

Schumpeter, Joseph A, [1954], "*History of Economic Analysis*", George Allen & Unwin, London 1954.

Studenski, Paul, [1958], *"The Income of Nations. Theory, Measurement, and Analysis: Past and Present*," New York University Press 1958.

Swamy, S, [1965], "Consistency of Fishers Test," *Econometrica, Vol. 33 No 3*, July 1965, p 619-623.

SOU, [1999], *"Konsumentprisindex*", Betänkande från utredningen om översyn av konsumentprisindex. Summary in English, SOU 1999:124.

SOU, [2001], "*Behovet av ekonomisk statistik*," SOU 2001:34, Delbetänkande av Utredningen om översynen av den ekonomiska statistiken, Stockholm 2001.

Wald, A, [1937], "Zur Theorie der Preisindexziffern," *Zeitschrift fyr Nationalökonomie* 8, p 179-219.

Walsh, C. M., [1901], *"The Measurement of General Exchange Value*", New York, McMillan.

Walsh, C. M., [1924], "Professor Edgeworth's Views on Index-Numbers", *Quarterly Journal of Economics* 38, 1924, pp 500-519.